Design methodology for axially loaded auger cast-in-place and drilled displacement piles

and

Load and Resistance Factor Design of drilled shafts in sand.

327. Fellenius, B.H., 2013. Design methodology for axially loaded auger cast-in-place and drilled drilled displacement piles. Discussion. ASCE ASCE Journal of Geotechnical and Environmental Engineering, 139(12) 2229-2231.

328. Fellenius, B.H., 2013. Load and Resistance Factor Design of drilled shafts in sand. Discussion. ASCE Journal of Geotechnical and Environmental Engineering, 140(1) 17014001-1 - 17014001-2

ASCE J of Geot. and Env. Engng, 140(1) 2229-2231

Discussion of Design methodology for axially loaded auger cast-in-place and drilled displacement piles. Sungwon Park, Lance A. Roberts, and Anil Misra, 2013. ASCE J of Geot. and Env. Engng, 138(12) 1431-1441.

Bengt H. Fellenius, P.Eng. Dr.Tech, M.ASCE
<Bengt@Fellenius.net>

The authors propose to employ t-z functions for estimating the load-movement response of a pile subjected to a static loading test. Judging by the authors' Fig. 1, the proposed t-z functions are "spring-slider" relations, that is, they model the shaft and toe resistances as bi-linear with an elastic initial part followed by a plastic part representing the ultimate resistance (capacity). The authors state that the capacity is mobilized at a pile-head movement of 5 % of the pile diameter, which definition they also apply to the pile toe load-movement response.

First, I would expect that the movement should be the movement at the pile element considered—a pile does not have to be very long before a substantial portion of the movement at the pile head is due to pile axial compression. It is not logical to assume that the length of the initial elastic response decreases with depth, all other factors being equal.

Second, applying an elastic-plastic model for the pile toe response is incorrect. The response of pile toe—usually denoted "q-z" function to separate it from the "t-z" function expressing the shaft response—does not show an ultimate resistance, but is a gently rising curve showing no kinks or explicit changes in curvature.

Third, the pile diameter has nothing to do with the changeover from elastic to plastic response, nor indeed, anything to do with the ultimate shaft resistance. The magnitude of the movement at the point of transition from elastic to plastic response may differ in different soils. However, it is usually also much smaller than the authors' mentioned range of 10 to 20 mm. Moreover, it is independent of the pile diameter, be the pile a small diameter pile, say 300 mm, or a large one, say 3,000 mm, and be the pile shape circular or square, or rectangular, such as a barrette, which can have one side about 1 m length and the other between 3 to 8 m length.

Fourth, the elastic-plastic response is too simplified a model for the shaft resistance. While it can occur, a strain-hardening response is more common, particularly in sand that includes silt and clay. Other soil types can exhibit strain-softening response, particularly in soft clays.

The mathematical expressions for four common t-z functions are given in Eqs. 1 through 4 (Fellenius 2012). Eq. 1 presents the relation for the "Ratio Function" so called because it states that the ratio between two values of unit shaft resistance (or load) is equal to the ratio between the movements produced by the same loads raised to an exponent, Θ.

Eq. 2 presents the equation for a hyperbolic relation of the stress as a function of the movement.

$$r_1 = r_2 \left(\frac{\delta_1}{\delta_2} \right)^{\theta} \qquad (1)$$

where
- r_1 = toe resistance for a point on the curve, e.g., Point 1
- δ_1 = movement for Point 1
- r_2 = toe resistance for a point on the curve, e.g., Point 2
- δ_2 = movement for Point 2
- Θ = an exponent

$$r = \frac{\delta}{C_1\delta + C_2} \qquad (2)$$

where
- r = force variable
- δ = movement variable
- C_1 = the slope of the line in a r/δ vs. δ diagram; the Chin-Kondner plot
- C_2 = ordinate intercept the r/δ vs. δ diagram

$$C_1 = \frac{1}{r_u} \qquad C_2 = \frac{\delta_1}{1/r_1 - C_1}$$

- r_1/δ_1 = any load/movement pair
- r_u = resistance occurring at infinite movement

Eq. 3 presents a relation for the ultimate (peak) resistance with the curve following the Hansen 80-% relation, which in words states that "the peak resistance is the stress that occurred for a movement that is four times the movement that occurred for the load equal to 80 % of the peak value". The 80 % function models a strain-softening response. It is not suitable for modeling the pile toe response.

$$r = \frac{\sqrt{\delta}}{C_1\delta + C_2} \qquad (3)$$

where
- r = shaft shear force variable
- δ = movement variable
- C_1 = the slope of the straight line in the √δ/r versus movement (δ) diagram
- C_2 = ordinate intercept of the straight line in the √δ/r versus movement (δ) diagram

$$C_1 = \frac{1}{2r_u\sqrt{\delta_u}} \qquad C_2 = \frac{\sqrt{\delta_u}}{2r_u}$$

- r_u = ultimate resistance
- δ_u = movement at ultimate resistance

$$r_u = \frac{1}{2\sqrt{C_1C_2}} \qquad \delta_u = \frac{C_2}{C_1}$$

Eq. 4 presents a relation proposed by Vander Veen (1953), which is useful for expressing elasto-plastic response.

$$r = r_u(1 - e^{-b\delta}) \qquad (4)$$

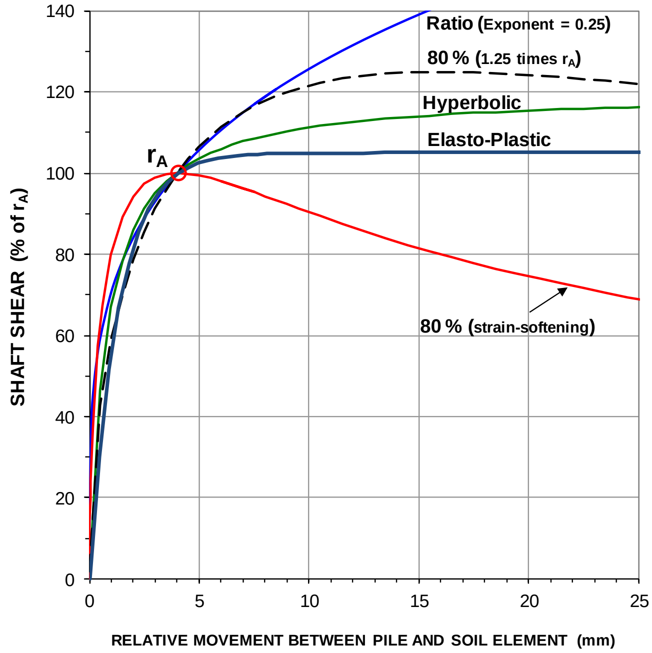

Fig. 1 Typical t-z functions produced by Eqs. 1 through 4

where	r	=	shaft shear force variable (or toe stress)
	r_u	=	ultimate resistance
	δ	=	movement variable
	b	=	coefficient
	e	=	base of the natural logarithm = 2.718

Using the equations to fit a theoretical curve to measured stress-movement curve is simple. Figure 1 shows curves developed from the equations assigning the curves to go through a common point, rA, at a movement value of 4 mm and an abscissa-value of 100, say 100 % of stress, load, force, whatever. The assigned point determines the curve for the 80 % function. Due to the interrelation between the factors expressed in the other equations, adding only one new parameter is sufficient to determine each curve. Choosing a peak at 15 mm movement for 1.25 times the 100-load (the 100 then becomes the 80-% value) further demonstrates the strain-softening connotation of the 80 % function. The curves can easily be reproduced in a simple spread-sheet calculation from the equations or by the UniPile program (Fellenius and Goudreault 1999).

When back-calculating a shaft resistance stress-movement response measured in a static loading test, I have found that a good fit to the measured values can be obtained by one or two of the four equations. It is often quite difficult to say beforehand—to predict—which function would deliver the best fit. Usually, the final fit shows that the elasto-plastic (or elastic-plastic) response is the least suitable relation. For pile toe-resistance, most often the best fit is achieved with the Ratio Function, while never with the elasto-plastic or the strain softening functions. Of course, as the pile toe load-movement curve is much flatter for the toe as opposed to the pile shaft curve, the particular parameters to input in the calculations will not be the same as those used to model the shaft resistance response. Also the Hyperbolic Function can be suitable for modeling the pile toe response, in particular for a pile subjected to residual load at the pile toe.

Often the results of a static loading test are only available as load and movement for the pile head up to a maximum load or to a load perceived as the "ultimate" resistance. To simulate that curve by modeling a pile as a number of elements affected by a shaft resistance response corresponding to a series of t-z functions and a pile toe response corresponding to a q-z function is relatively simple process (Fellenius and Goudreault 1999). However, as in every modeling of a process that reacts to three or more parameters, a final good fit can be a result of compensating errors in the choice of parameters and assumptions. Then, a good fit does not prove anything. The only assured way is to fit calculated t-z curves to loads and movements measured at individual pile elements in an instrumented pile (e.g., Fellenius and Nguyen 2012). When the so-fitted individual elements are integrated in simultaneous calculation to produce a simulated pile-head load-movement curve that shows to agree reasonable well with the measured curve, then, one can be justified in assuming that the various soil parameters and t-z curves are representative for the site and piles and expect that a theoretical calculations of another pile would serve as a reasonable prediction.

As a final point, the sum of the peak resistance for all elements will not be the same value as the "ultimate" resistance determined from the pile-head load-movement curve, because at an applied pile-head load equal to the ultimate resistance evaluated from the curve, the peak resistance has been passed and the upper elements will have a resistance value that is smaller (strain-hardening) or larger (strain-softening) than the sum of their individual peak values. Note also that in strain-softening soils, the pile-head load-movement curve may indicate a post peak response even when the pile toe resistance is increasing, as it always does.

While I wholeheartedly agree with the authors that evaluation of measured a pile response and design of piled foundations should be based on the load-movement response expressed in t-z and q-z functions, I find the authors' "spring-slider" function somewhat rudimentary, however, and suggest that more developed functions be employed, as detailed in this Discussion.

References

Fellenius, B.H., 2012. Basics of foundation design, a text book. Revised Electronic Edition, [www.Fellenius.net], 384 p.

Fellenius, B.H., and Nguyen, M.H., 2013. Large diameter long bored piles in the Mekong delta. International Journal of Case Histories, 2(3) 196-207.

Goudreault, P.A. and Fellenius, B.H. (1999). UniPile Version 4.0 for Windows, User Manual. UniSoft Ltd., Ottawa, [www.UnisoftLtd.com], 64 p

Van der Veen, C., 1953. The Bearing Capacity of a Pile. Proceedings of the 3rd ICSMFE, Zurich, Switzerland, August 16-27, Vol. 2, pp. 84-90.

ASCE J of Geot. and Env. Engng, 140(3)

Discussion of Load and Resistance Factor Design of drilled shafts in sand. D. Basu and Rodrigo Salgado. ASCE J of Geot. and Env. Engng, 140(3) 17014001-1 - 17014001-2.

Bengt H. Fellenius, P.Eng. Dr.Tech, M.ASCE
<Bengt@Fellenius.net>

The authors relates their approach to Load and Resistance Factor Design, LFRD, of drilled shafts in sand on three definitions: pile capacity determined in a static loading test, limit unit shaft resistance", and limit pile base resistance, alternatively expressed as "ultimate" toe resistance.

First, the authors define pile capacity determined in a static loading test as the load that induces a pile head movement equal to 10 % of the pile diameter. The definition has its origin in a mistaken quotation of a now 70 year old statement by Terzaghi (1942). Terzaghi wrote: "*the failure load is not reached unless the penetration of the pile is at least equal to 10 % of the diameter at the tip (toe) of the pile*". For full quotation and context, see Likins et al. (2012). Note, Terzaghi did not define the capacity as the load generating a movement equal to 10 percent of the pile head diameter, he emphatically stated that whatever definition of capacity or ultimate resistance used, it must not be applied until the pile toe has moved at least a distance corresponding to 10 percent of the pile toe diameter. (The pile head will then have moved an additional distance equal to the pile compression).

Most certainly, Terzaghi did not suggest that a fixed pile-head movement value, however determined, could serve as a definition of capacity. A definition based on a value of absolute movement of the pile head disregards the effect of pile compression, which can be a significant part of the pile head movement. A short pile will undergo only small compression for the applied load, whereas a long pile will have a larger pile compression.

Most full-scale static loading tests where the pile shaft resistance is essentially mobilized are carried out to pile head movements in the range of 10 mm through 30 mm. For some of these tests, the pile-head load-movement curve may trend to progressively increasing movement for increasing load, implying a total pile capacity as interpreted by some definition or other. Basing a capacity definition on the load-movement curve or curvature is satisfactory for an engineering application. A distinct movement value could only be satisfactory for use as a limit if it refers to a value determined from what is acceptable for the foundation supported on the piles. In my opinion, a definition such as that suggested by the authors is not relevant for the LRFD.

Second, the authors assume that the pile toe response exhibits an ultimate resistance. However, there is no such thing as an ultimate toe resistance. The load reaching the pile toe will result in an increasing pile toe movement with no distinct trend change that could be taken to be representative for an ultimate resistance. This fact has been well-established in full-scale tests on instrumented piles with direct measurement of pile toe load (stress) response. Figure 1 shows two pile toe load-movement responses, so-called q-z functions, typical for the pile toe response. The curve denoted "Ratio" follows Eq. 1 which expresses the stress-movement as the ratio of any two resistances set equal to the ratio of the respective movements raised to an exponent. The curve denoted "Hyperbolic" follows Eq. 2 is a hyperbolic relation of the stress as a function of the movement.

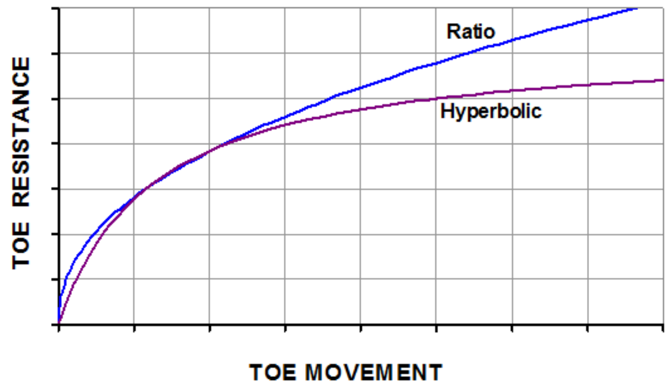

Fig. 1 Two typical pile-toe load-movement curves: q-z functions

$$r_1 = r_2\left(\frac{\delta_1}{\delta_2}\right)^{\theta} \qquad (1)$$

where
r_1 = toe resistance for a point on the curve, e.g., Point 1
δ_1 = movement for Point 1
r_2 = toe resistance for a point on the curve, e.g., Point 2
δ_2 = movement for Point 2
Θ = an exponent

$$r = \frac{\delta}{C_1\delta + C_2} \qquad (2)$$

where
r = force variable
δ = movement variable
C_1 = the slope of the line in a r/δ vs. δ diagram; the Chin-Kondner plot
C_2 = ordinate intercept the r/δ vs. δ diagram

$$C_1 = \frac{1}{r_u} \qquad C_2 = \frac{\delta_1}{1/r_1 - C_1}$$

r_1/δ_1 = any load/movement pair
r_u = resistance occurring at infinite movement

A pile toe that would respond to load by a curve not qualitatively similar to those shown in Figure 1 would be affected by some special conditions, such as residual load causing a change of curvature due to the fact that the pile toe initially is in a re loading mode, or because the soil around the pile toe is cemented with the cementation breaking down as the load increases, or other non-continuous influence causing sudden change of response. Of the two q-z functions shown in the figure, I have found the Ratio Function to be the one that best models the pile toe response from small to a large toe movement.

Third, regarding pile shaft response to load, the authors' equation for the unit shaft resistance of a pile-soil element includes four parameters: the "relative density", DR, (density index, ID, in international standard), the coefficient of earth stress at rest, K0, the tangent of the soil friction angle called "triaxial-compression critical-state friction-angle", Φc , and a coefficient governed by angularity, C1.

The Density Index is the ratio between (e_{max} - e_{actual}) and (e_{max} - e_{min}) and it is a highly imprecise and non-reproducible parameter. A void ratio value determined on a sand sample is usually provided with two-decimal precision. However, the value is rarely more precise than by about 0.05±. For loose to compact uniform sand, the in-situ void ratio values typically range from about 0.20 through 0.60. Therefore, the ID for a given sample, say, with an in-situ void ratio of 0.40, where typically, the maximum and minimum void ratios lie between 0.30 and 0.70, the ID is 75 %. However, considering an error of 0.05 up or down for each of the three values, the error in a particular ID could be almost 20 %. Tavenas and LaRochelle (1972) presented a detailed study of the Density Index and indicated that the average error is 18 % and concluded that the index "cannot be used as a base parameter of any calculation".

The coefficient of earth stress as rest, K_0, before piles are installed is often assumed to be about 0.5. Where piles have been driven into the sand, the associated vibrations and displacement will usually increase the earth stress acting against the pile and the new earth stress coefficient can exceed unity. In dense overconsolidated sand, where typically the in-situ K_0 is greater than unity, the sand can actually become looser than before the pile driving and the K_0-value diminishes. (Usually, however, there would be little need for piles in such sands).

I do not know how large the usual imprecision is for the "triaxial-compression critical-state friction-angle". However, I would expect that it would be overoptimistic to expect a precision better than one degree, which means an error of about 5 % in the $\tan\delta$ or $\tan\phi$.

According to the authors, the coefficient to adjust to the angularity of the sand can range from 0.63 through 0.71, i.e., a range of about 10 %. Determining sand angularity is rarely a part of the usual site investigation, however.

Moreover, the authors' equation for unit shaft resistance does not include any similar adjustment for mineral composition. For example, the shear response of silica sand is quite different to that of calcareous sand, and the shear response of sand containing just a few percent of mica will have a less stiff shear response compared to sand with no mica content. Moreover, sand containing fines will show a different response to that of uniform (clean?) sand.

Naturally, the uncertainties the parameters do not all work in the same direction. However, in my opinion, in an actual case, the combined effect of the errors will result in a non-trivial total error of the unit shaft resistance evaluated from the authors' equation. Moeover, the imprecisions in the relation for the shaft resistance and the assumption of an ultimate toe resistance raise questions on the practicality of using the authors' shaft and toe relations for assessment of the LRFD approach.

References

Likins, G.E., Fellenius, B.H., and Holtz, R.D., 2012. Pile Driving Formulas—Past and Present. ASCE GeoInstitute Geo-Congress Oakland, March 25-29, 2012, Full-scale Testing in Foundation Design, State of the Art and Practice in Geotechnical Engineering, ASCE, Reston, VA, M.H. Hussein, K.R. Massarsch, G.E. Likins, and R.D. Holtz, eds., Geotechnical Special Publication 227, pp. 737-753.

Tavenas, F. and LaRochelle, P., 1972. Accuracy of relative density measurements. Geotechnique, 22(4) 549-562.

Terzaghi K., 1942. Discussions on the progress report of the committee on the bearing value of pile foundations. Proceedings of the American Society of Civil Engineers, Issue of February 1942, pp. 311-323.